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Does Domain of Absolute Value Function Always Positive? Commognitive-Conflict Analysis of Students of First Semester

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Abstract

This research aims to analyze the result of students' work in finding the solution to math-problems about absolute functions. Commognitive's analysis was used to analyze student works in this qualitative research. This research's methods were, namely, preparing, collecting data, transcription data, and analyzing data. A total of 25 students from two universities were given math questions, and two were selected as research subjects. The selection of subjects is based on different and exciting work results. The research subjects attended in-depth interviews to learn more about the commognitive's components. The results showed that commognitive conflict occurred in the word use component when the subject experienced errors in interpreting the data on the questions. Subjects cannot correctly describe the graphic requested in the questions (visual mediators). Describing what is known in the problem is also incompatible (narratives). Subjects experienced the incorrect routines. Based on the results of in-depth interviews, it can be said that the commognitive conflicts occurred because of a lack of understanding of the concepts.

Keywords Commognitive Analysis; Commognitive Conflict; Absolute Function

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1. INTRODUCTION

Thinking and communicating in Mathematics are inseparable (Sfard, 2014, 2016, 2018). Communicating orally and in writing is essential in teaching and learning activities. In those activities, communication occurs in mathematical discourse(Thoma & Nardi, 2018). In this discourse, the interlocutor can raise the same or different arguments. This different situation is stated as a commognitive conflict(Putut et al. l., 2023). In the first year of lectures, students study various subjects, including algebra, number theory, calculus, and other subjects. Each of these courses has the potential for cognitive conflict. In calculus courses, for example, absolute value functions have the potential for conflict. The difference in understanding the absolute value function in high school and university triggers a conflict(Emanuel & Meilantifa, 2022; Putut, Emanuel, & Anam, 2022; Putut et al., 2023). The assumption that absolute value is always non-negative leads to conflicts when receiving lecture material in the first year.

In a number theory course, understanding the division of integers also has the potential for commognitive conflict(Putut et al., 2023). Commognitive conflict is a condition in which what is in the mind differs from what is being communicated (Ioannou, 2017, 2018). Commognitive is the lens to view a thought process which is communication with oneself in both verbal and symbolic forms (Presmeg, 2016). Commognitive conflicts can occur at the high school to university level in various Mathematics topics, including functions, trigonometry, opportunities, and many more(Ioannou, 2016, 2017, 2018; Viirman, 2014, 2015; Viirman & Nardi, 2019a). Because of that, however, the topic of function is one of the most important things to learn in the first year of the lecture because it is helpful for later years, and because of that, we chose this research.

Commognitive theory states that discourse comprises four components: word use, visual mediators, narratives, and routines(Scott, Hargreaves, & Sfard, 2015). Words use refers to colloquial and literate words (Lu, Zhang, & Stephens, 2019). Visual mediators could be graphics or charts (iconic) and specific symbols (symbolize) that were used in works(Fernández-León et al., 2019). Narratives are arguments memorization or substantiation that could be spoken and written, describing processes and the relationships between processes, such as definitions, theorems, and proofs(Sfard, 2020). Routines are the iterative patterns of existing discourses that could be ritualization or exploratory (Presmeg, 2016).

Based on components, mathematical communication is very dependent on the type of interlocutor, such as the use of words(Scott et al., 2015; Tabach & Nachlieli, 2016). The emergence of differences in the use of words used by interlocutors is a big challenge in mathematical communication and is one type of challenge that is commognitive conflict(Pratiwi et al., 2020). Commognitive conflict occurs when there are differences in the use of discourse between students and teachers (Pratiwi et al., 2020; Sfard & Avigail, 2007; Thoma & Nardi, 2017, 2018; ZAYYADİ, 2020). Commognitive conflict encourages changes in defining words or identifying numbers(Kim et al., 2019a; Zayyadi et al., 2019). Finally, students can accept a new discourse(Ioannou, 2018; Kim et al., 2019b; Nardi, Ryve, Stadler, & Viirman, 2014; Viirman & Nardi, 2019b). Because of that, commognitive conflict is the most important topic for research.

Research on commognitive conflict analysis in mathematical discourse has been done(Kim et al., 2019a; Pratiwi et al., 2020; Thoma & Nardi, 2017, 2018; Viirman & Nardi, 2019b; ZAYYADİ, 2020). Analysis of commognitive conflicts in learning and teaching calculus happened in the first year of the lecture(Nardi et al., 2014). In learning subgroup theory, commognitive conflicts had founded(Ioannou, 2018). Commognitive conflict regarding the routines component also happened in students' answers to practice questions(Thoma & Nardi, 2018). They analyzed the results of first-year student answers where commognitive conflicts that had not been resolved until the final exam were viewed from word use and routines. Four components of commognitive were used to analyze students' answers in high school from the given mathematical problems(Zayyadi et al., 2019).

In the material on limits of absolute value functions, for example, the conflict occurs when the subject expresses his opinion that absolute value is always positive(Emanuel & Meilantifa, 2022). This happened during a semi-structured interview, and he expressed this opinion. When asked what about zero is positive, the subject felt confused and doubtful, so he became worried about the truth of his opinion. The subject can finally accept that his opinion is wrong, and he states that absolute values are always non-negative. The commognitive conflict occurs here between the researcher and the subject in the semi-structured interview. Still on the same topic, namely limits of absolute value functions, a conflict occurs when the subject states that function limits do not exist because the function is not continuous(Ngin, 2018). When further information was explored through interviews, it was found that the subject had old knowledge about continuous functions, contrary to the applicable rules. When the subject tries to visualize using function tables and graphs and then believes that function limits do not exist, there is also an indication of commognitive conflict. Through semi-structured interviews conducted by researchers, it was discovered that there was a commognitive conflict in the visual mediator component, namely the use of discrete tables, which conflicted with the functional domain, namely the set of real numbers. The resulting function graph also contradicts the function domain because it is a continuous graph, whereas the function table whose graph depicts it should be in the form of points and not connected continuously. At the end of that research, the conflicts were not solved. This research on the analysis of student work results in solving mathematical problems on function from a commognitive point of view has been carried out but only focused on word use. However, research on commognitive conflict about function is still needed, especially about absolute function. This study analyzed student work in terms of four commognitive components. The results of this study can provide clues for the following research on other materials and help reduce or eliminate commognitive conflicts in students in their first year of study.

2. METHODS

This research was qualitative and well-designed. Researchers, as the main instrument used qualitative research methods to describe data as it is according to what is found in the field (Cresswel, 2013). The qualitative design explores the four commognitive components: word use, narratives, visual mediators, and routines. This study involved 25 students from two private universities, namely the Islamic University of Malang and the University of Wijaya Kusuma Surabaya. The instrument in this study was a written test consisting of one question about function. Students are asked to work on the problem for 20 minutes and write down their steps. The given math problem allows for commognitive conflicts to arise.

The instruments provided in this study consisted of a question:

"Given $f(x) = |x-3| + |2x+1|$, where $x \in R$. What is the domain division of this function? Can you find the set of solutions for the equation $f(x) = 5$? Explain your answer!"

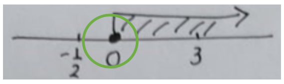
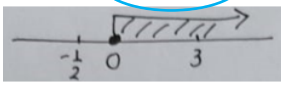
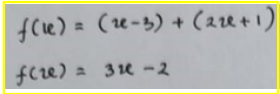
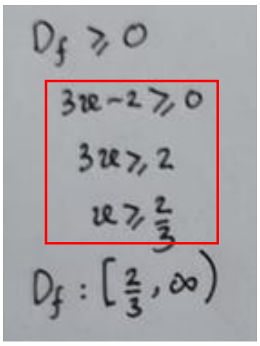
From the 25 students who collected answers, two students were indicated to have a commognitive conflict based on the difference in their answers. Based on their works that indicated conflicts, they were selected for a more in-depth interview to obtain data on commognitive conflict, analyzed by the four commognitive components. The primary data used in this study are the results of students' work. In-depth interviews conducted were used as data triangulation in the study.

3. FINDINGS AND DISCUSSIONS

The results of the answers of 25 students were grouped based on the presence of indications of commognitive conflict. A total of 10 students indicated a commognitive conflict, while 15 had none. From the grouping results, two students who indicated commognitive conflict were selected as research subjects, namely P1 and P2, for in-depth semi-structured interviews. This section will explain the differences between the two subjects (P1 and P2) in the troubleshooting function steps. Subject P1 took several steps in solving the problem. Subject P1's answer was confirmed through in-depth interviews with no intervention to the subject. Table 1 below is the results of interviews with P1 subjects based on the results of their work.

Table 1. Interview regarding confirmation of work on subject P1

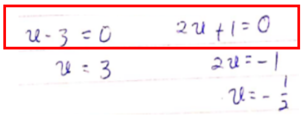
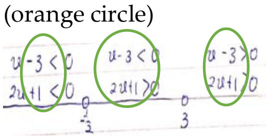
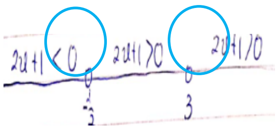
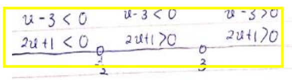
Transcripts	Words use	Visual mediators	Narratives dan Routines
R: Your answer to number 2 is exciting and unique. Please explain how the process is done.			
P1: Yes, sir. Um ... first, I rewrite the function given in the problem.	P1 looked at their work.		

P1: Next, I draw a number line consisted $-\frac{1}{2}$ and 3. After adding zero, I shaded the area from zero to the right. Next, I omitted the absolute value sign and got a simple function.	P1 wrote numbers words.	P1 drew below  P1 shading areas greater than equal to zero, and that is wrong (blue circle) 	P1 suddenly obeyed the absolute value sign, which is false(yellow rectangle).  Set domain f more than equal to zero, and this does not seem right. Solve for the linear inequality and represent the domain f.
R: Why did you make that decision? Give the reason!			
P1: I thought that $\|x-3\| \geq 0$ and $\|2x+1\| \geq 0$. So, I omitted the absolute mark. I shaded the area on the number line for x greater than or equal to zero.	P1 watched his work	P1 draws a number line, and the shaded area	The argument for P1 to omit the sign of the absolute value is based on a lack of understanding of the definition of the absolute value, so the step is wrong.
P1: The next step is because the domain of the function is more than or equal to zero, then I get $x \geq \frac{2}{3}$. So, the domain of function $[\frac{2}{3}, \infty)$	P1 resolves the inequality but is incomplete in the use of signs, i.e., between the first and second lines, there are fewer signs ⇔ , so it is not correct (red rectangle) 		P1 takes the steps to solve the inequality. The P1 argument regarding the function's domain indicates an error in understanding the concept, so the job is wrong.

Subject P2 completed the question in several steps. The table below shows the depth-interview confirmation of the processing of questions by P2.

Table 2. Results of the interview confirmation of subject P2

Transcripts	Words use	Visual mediators	Narratives dan Routines
R: Your work is exciting. I am very interested. Can you			

explain the process of working on that matter?			
P2: Um, okay, sir. Well, question number 2 is an absolute value. The function $f(x)$ consists of two absolute values. So, I first determine the limits of the interval for each absolute value, that is, by finding the value of x, they equal to zero, so that I get the x value (red box)	P2 pays attention to the problems he has worked on. P2 writes down notations, variables, symbols, numbers.		P2 writes a simple equation for determining the value of x, equating to zero, which is wrong(red rectangle).  P2 solves the equation and gets the value for x, two values of x are obtained.
R: Why do you equal zero?			
P2: Because I remember that from the definition of absolute value, ehm… what… for the limit, it is found that the absolute value function is equal to zero. That is what I think, sir.			P2 argues that to determine the limit of the absolute value function equals zero and that is wrong.
P2: After that, I plotted it into the same number line. The limits are $-\frac{1}{2}$ Moreover, 3 are not members of the domain pack. Left $-\frac{1}{2}$ all are less than zero. Between $-\frac{1}{2}$ to 3, for $2x+1$ is more than zero, and $x-3<0$. Meanwhile, for $x>3$, both are more than zero. I see, sir.	P2 uses the > and < signs on the number line, and that is wrong because there should be ≥ (orange circle) 	P2 draws a number line to define the area with a wrong circle (blue circle). 	P2 states that -1/2 and 3 are boundaries, and the unmembered domain is wrong. P2 repeats the use of the "> " sign (when $2x+1>0$) by writing $2x+1>0$ and $x-3>0$, this is wrong. Repeat the solving steps for linear equations without using the equivalent sign, which is wrong (yellow rectangle). 

The results of the work of the two subjects, as well as the interviews conducted, show the process of their work on solving the problems to confirm their work through commognitive analysis. The next was the analysis using a commognitive lens.

Words use

The two subjects experienced commognitive conflict from the component's view of the word use component. They showed that word use was beneficial in mathematical discourse. This agrees with the argument that students use words (including notation, number symbols, and number operation signs) in mathematics to solve problems. P1 used the number symbols, number operation signs, and notation correctly. P1 did not use the equivalent sign when solving the existing equations, which is wrong. The first step to the next step should use the equivalent sign. Interviews were conducted to confirm this, and obtained information that P1 considered the incident normal. The researcher then conveyed to the P1 that there must be a relationship between the lines, marked with an equivalent sign in solving an equation. Here, there is a conflict in P1 regarding the word use component. The P1's lack of understanding of the use of the equivalent sign in solving equations is the cause of commognitive conflict.

However, while P2 wrote in the number line image, there was a difference. Using symbols is more excellent than using symbols greater than equal. Based on the semi-structured depth interview, P2 assumed that the absolute value is never negative, and the value of x obtained from the absolute value component is zeroed only as a limit so that it does not become a member of the domain, which is wrong. The insufficiency of understanding of subjects on the theory of the absolute value function is the cause of the conflict here.

Visual mediators

P1 described a number line according to the problem given first. Based on what is given in the questions, P1 determined that there were two values $-\frac{1}{2}$ Moreover, 3, which lay on the number line. Then, P1 added zeros to the number line, shading the area to the right of zero. The addition of zeros to this number line was confirmed through in-depth interviews. Based on the results of the interview, it was found that the reason why P1 added zero to the number line meant that the absolute value was always non-negative, so $|x-3| \geq 0$ and $|2x+1| \geq 0$, then P1 wrote zero on the number line. The next step was P1 shaded the area to the right of zero, which was wrong. The researcher tried to remind P1 of the absolute value concept to the subject, and after recalling the concept of absolute value, the subject realized that what he had done was wrong.

P2 worked on the problem by drawing a number line containing $-\frac{1}{2}$ and 3. P2 added an 'o' sign to the picture when drawing the number line. The 'o' indicated that the number was not included in the function's domain and was wrong. In clarification by interview, P2 considered that the number was only a limit, so it did not become the domain of the function.

Narratives

Commognitive conflict appears shown in the explanation such as arguments of the strategy taken by the subject when solving the problem. When working on the problem, the absolute value sign was suddenly omitted, which was wrong. P1 specified that the domain was more significant than or equal to zero, which was false. Pointed on the interview, it was found the reasons why P1 took the action. P1 considered that the absolute value function was always non-negative, so there was no problem if the absolute sign was removed because the result was never negative. This matter was deepened again during the interview, and the lack of understanding of the absolute value function caused this conflict.

Another subject, P2, began the problem-solving step by equating zero to the component of the function $f(x)$. P2 did this to get the x value, and this step was wrong. After being confirmed through interviews, P2 stated that this step was taken to find the limit of the x value on the number line. P2 also argued that the boundary was not a member of the function's domain, which was wrong. The commognitive conflict that occurs here after trying to find the cause, it turns out that a lack of understanding of the concept of the absolute value function is still the cause. Commognitive conflict occurs due to errors related to understanding the given problem and using sentences in solving problems.

Routines

The ability to explore thinking can be identified from a person's ability to use multiple methods to find solutions to problems. P1 suddenly omitted the absolute value sign and performed a function addition operation, which was wrong. Set the function's domain f greater than or equal to zero, which was false. The step in solving the linear inequality to determine the domain of the function f and repeating it when solving the equation without using the equivalent sign was also wrong. From the interviews between the researcher and the subject, it can be found that the concept of the concept of the absolute value function by the subject was still poor and caused the conflict.

P2 was repetitive when determining the function's domain, namely writing $2x + 1 > 0$ in the middle, which is repeated by writing $2x + 1 > 0$ and $x - 3 > 0$ on the right, which was wrong. P2 also repeated steps when solving linear equations by not using the equivalent symbol, which was wrong. Researchers confirmed the results of this work to the subject through interviews, and it was found that the cause of this conflict was the subject's understanding of the value of $x = 3$ and $x = -\frac{1}{2}$ was only a boundary and not a member of the absolute value function domain. This shows that the understanding of the concept of the absolute value function was still lacking, causing a commognitive conflict here. Subjects experienced commognitive conflict in this study because they were wrong in choosing strategies for problem-solving. The strategy that had previously been in their minds. The following will discuss in more depth the commognitive conflicts that occur in research.

Discussion

Several commognitive conflicts happened in the transition from secondary to tertiary school. It dealt with the research by Thoma Nardi (2018). Specifically, the commognitive conflict that we served to introduce a concept (in our case of a function of absolute value). Hence, deducing general characteristics from examples instead of particular aspects is an objective founded in this transition period (Corriveau & Bednarz, 2017; Rach & Ufer, 2020) rather than in the school stages. Commognitive conflict with concrete, worked-out examples and exercises also appear in the transition stage. We have just observed in our results, it can affirm that they are more common in secondary school since students at this stage are less capable of inferring general characteristics from several examples. Students have a tendency not to think critically and are too lazy to develop the knowledge they have acquired(Aizikovitsh-Udi & Cheng, 2015; Chikiwa & Schäfer, 2018; Fazriyah et al., 2018; Hafni, 2017). This causes their knowledge not to develop, and ultimately they do not understand the material being studied(Ampuero, Miranda, Delgado, Goyen, & Weaver, 2015; Erikson & Erikson, 2019; Heinrich, Habron, Johnson, & Goralnik, 2015; Jones, 2017; Larsson, 2017). The steps taken are still based on knowledge gained but not developed so that when they encounter new problems, they will experience errors in solving the problems(Heinrich et al., 2015; Mumford et al., 2017; Widana, 2018). Using words used when solving questions can also result in stating different answers. Apart from that, arguments that are based on fragile knowledge mean that the reasons put forward can also be wrong(Im, Yoon, & Cha, 2016; Kaya & Aydin, 2016; Meyer, 2018; H. J. Park et al., 2016; J. H. Park & Lee, 2018; Son, Han, Kang, & Kwon, 2016; Wu & Perng, 2016). All of that is happened in the mathematical discourse.

Sfard (2018) claims that discourse is structurally "as objects and operationally as processes". She also said that the analysis of the discourse taught us how a student perceives an object. In the case of processes, the discourse expresses actions and is personalized, while in the case of objects. In our case, we can see that subjects start the lesson with procedural definitions (i.e., describing how to construct an absolute value) instead of structural definitions (i.e., describing properties that characterize an object) (Kobiela & Lehrer, 2015). These types of procedural definitions with generic examples justify using the remaining examples (concrete, worked-out examples, and exercises) to consolidate the use of the procedures or become familiar with the notation. However, concrete examples also appear at the tertiary level, where structural definitions are used to prove that a specific object that we have defined is not the empty set (Martín-Molina, González-Regaña y Gavilán-Izquierdo, 2018). Commognitive conflicts are happened at school than at university(Kim et al., 2019a; Lu, Tao, Xu, & Stephens, 2020). This is because most students prioritize the results rather than the problem-solving process. Problem-solving steps are sometimes not given much attention. Students' poor understanding of concepts can cause conflict(Thoma & Nardi, 2017).

Furthermore, Nachlieli Heyd-Metzuyanim (2021) claims that school mathematical knowledge and the connections between their concepts are critical factors of success in the exams of a first-year university course. Finally, commognitive conflicts referring to the difficulty help eliminate students' prejudices about their mathematical skills and motivate them to complete those tasks. Indeed, in line with Halim et al. (2020), it was ensured that students who perceive mathematics as very difficult tend to abandon the tasks having made less effort than those who consider it easy. We are convinced that they are more common in the preceding courses of transition because of their focus on students instead of content.

4. CONCLUSION

In mathematical discourse, commognitive conflicts occur in solving various problems, including algebra, opportunities, sets, and other topics. Commognitive conflict can occur in one or more commognitive components: word uses, visual mediators, narratives, and routines. It could happen altogether. In the mathematical discourse regarding functional problems, commognitive conflict appears in the review of the four components: words use, routines, narratives, and visual mediators. The results of in-depth interviews with research subjects show that the student's lack of understanding of function causes commognitive conflict. This provides an opportunity to conduct further research to eliminate commognitive conflicts that occur in students.

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