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Construction of Integer Operations of the Gasing-Mathematics Method from the Perspective of APOS Theory

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Abstract

A low mastery of integer operations remains a significant cause of students' weak mathematical literacy, as current teaching methods often emphasize mechanical computation over conceptual understanding. This study aims to describe how students construct their understanding of integer operations through the Gasing Method from the perspective of APOS Theory. This research employed a descriptive qualitative design with a task-based case study approach, involving three mathematics education students who had participated in Gasing training and obtained the highest scores on an integer operation test. Data were collected through written tasks and semi-structured interviews, then analyzed using the stages of APOS thinking: Action, Process, Object, and Schema, through iterative coding and triangulation. The results show that the Gasing Method supports the progression of students' thinking from procedural to conceptual: the Action stage appears in step-by-step computation, the Process stage in mental prediction and verbal explanation, the Object stage in recognizing relationships among operations, and the Schema stage in integrating these into a coherent conceptual framework. The study concludes that the integration of Gasing and APOS provides a powerful pedagogical model that bridges concrete and abstract reasoning, offering meaningful implications for mathematics teachers in designing culturally grounded yet cognitively informed numeracy learning.

Keywords Action; Gasing; Object; Process; Scheme

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1. INTRODUCTION

Integer operations are a fundamental topic taught across all levels of mathematics education. Although individuals often encounter integer operations unconsciously in daily life, students frequently struggle with these operations when they are formally presented in school mathematics. The complexity of mathematical symbols and the abstract nature of integer operations cause students to perceive this topic as confusing and intimidating (Srintin et al., 2019; Zaleha, 2018). Consequently, the level of numeracy among Indonesian students remains low. The Programme for International Student Assessment (PISA) reported Indonesia's average mathematics score as 379, which declined further to 366 in PISA 2022. This score is far below the OECD average of 472, with less than one percent of students achieving proficiency levels 5 or 6. These data indicate a persistent gap between curriculum objectives and actual student achievement in numeracy and conceptual understanding.

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Various strategies have been developed to improve student understanding of integer operations, including the use of technology-based learning media. However, research has shown that the mere integration of technology does not automatically enhance mathematical understanding. Found that students who frequently use digital devices in class do not necessarily perform better in mathematics; the effectiveness of technology depends greatly on pedagogical design and cognitive engagement. Therefore, approaches that rely solely on technology without addressing cognitive processes have limited success in strengthening conceptual understanding.

One innovative approach that has gained attention in Indonesia is the Gasing Method (*Gampang, Asyik, dan Menyenangkan*), developed by Yohanes Surya. The Gasing Method emphasizes enjoyable, concrete, and structured learning experiences that guide students gradually from real-world representations to abstract thinking (Hariati et al., 2020; Kusuma et al., 2018; Langi et al., 2021; Sary & Ristiana, 2019). This method aims to reduce students' anxiety about mathematics and promote a deeper understanding through the use of games, songs, and interactive problem-solving. Despite its popularity, most studies on the Gasing Method have focused primarily on learning outcomes such as speed and accuracy, rather than on the underlying cognitive processes that shape students' mathematical understanding.

To analyze such cognitive processes, this study adopts the APOS Theory, a constructivist framework developed by Ed Dubinsky and colleagues. APOS stands for Action, Process, Object, and Schema, representing the stages through which learners internalize mathematical ideas. An *action* is a manipulation of mathematical objects guided by external instructions (Langi' et al., 2023; Langi' et al., 2019; Oktaç et al., 2019). Through repeated actions, learners internalize and coordinate them into a *process*. When the process is reified into a comprehensive mental entity, it becomes an object, and ultimately, the schema emerges as a coherent structure that links various actions, processes, and objects. The APOS framework provides a systematic way to describe how understanding evolves from procedural to conceptual, which is crucial for analyzing students' cognitive construction in integer operations.

However, in the Indonesian context, the application of APOS theory remains limited and is rarely combined with culturally grounded or practical teaching methods, such as Gasing. Most existing studies treat these two approaches separately, APOS as a cognitive model, and Gasing as a pedagogical technique, without exploring their integration (Diah & Siregar, 2023; Jusmaniar et al., 2024; Rizaldi et al., 2023). The absence of this synthesis constitutes a significant research gap. This study, therefore, proposes to bridge that gap by analyzing students' construction of integer operation concepts using the Gasing Method from the perspective of APOS theory.

The novelty of this study lies in its integrative approach that combines the practicality of the Gasing Method with the cognitive depth of APOS theory. Unlike previous studies that merely measured learning outcomes, this research focuses on students' mental structures and how they transition through the stages of action, process, object, and schema while using the Gasing Method (Mulyawati & Sarwinda, 2020; Nga et al., 2023; Prasetyo et al., 2020). Such integration not only provides a new perspective on how local pedagogical innovations can be aligned with modern learning theories but also contributes to the development of a more comprehensive model of mathematics learning.

The purpose of this research is to describe how students construct their understanding of integer operations through the stages of APOS when learning with the Gasing Method. The research question guiding this study is: *"How is the construction of integer operations using the Gasing Method viewed from the perspective of APOS theory?"* Addressing this question will provide insights into the cognitive mechanisms underlying students' conceptual growth in mathematics.

The urgency of this research stems from Indonesia's ongoing struggle with low numeracy performance, which affects students' problem-solving abilities and competitiveness in the global digital era. The combination of the Gasing Method and APOS theory offers a meaningful and culturally contextualized framework for improving mathematics learning. This study not only aligns with the goals of Merdeka Belajar, which emphasizes active and meaningful learning, but also provides practical implications for designing instructional strategies that foster deep understanding rather than rote memorization.

Integer operations are a topic taught at every level of education (Prananda et al., 2021; Agbo et al., 2023; Ahmad et al., 2018). Unconscious recognition of integer operations has been developed in a person from an early age through various daily activities. However, building an understanding of integer operations, as packaged in mathematics learning in school, is not easy for students. Over the years, the integer operations that students have learned do not seem to have left a lasting impression on their memories. Many methods have been developed, including technology-based learning media; however, it has been found that they have not been able to facilitate students in building a thorough understanding of integer operations. This material is considered to be a complex collection of numbers with a dizzying process. As a result, the numeracy level of students and many community members is very low (Huang et al., 2017), and many students struggle to solve math problems quickly and accurately (Reflina, 2020). Then came the Gasing mathematics method, a breakthrough in the world of mathematics education that teaches mathematics in an easy, fun, and engaging way.

The Gasing method (easy, fun, and fun) is a mathematics learning method developed by Yohanes Surya, a mathematician from Indonesia (Adams, 2020; Ahmad et al., 2021; Clarke & Roche, 2018; Ghofur et al., 2022; Herlina et al., 2023). The background to the creation of math gaming is the tendency of students to have weak mathematical concept skills because they are afraid to learn mathematics (Batlolona et al., 2019). Mathematics is designed to arouse students' interest in learning by presenting learning material in a fun and engaging form, such as games or songs that convey mathematical concepts. In addition, gaining proficiency in mathematics allows students to learn the subject, starting from concrete to abstract stages, and ultimately reach the stage of quick calculation without writing (mental calculation). This allows for easier mastery of mathematics because the concepts learned are structured in a structured manner.

The Gasing method is a novel approach in mathematics. This method can be learned by all groups, including both young and adult individuals, and is tailored to the stages of cognitive development at each level of development. Many researchers have begun to examine this method, but they have primarily focused on achieving student learning outcomes using the tops method (Fitrieyah, 2021; Pertiwi et al., 2025), without paying attention to how the thinking process works when constructing an understanding of the tops method. One of the constructivist theories in mathematics learning is the APOS Theory (Action, Process, Object, and Schema). This theory was introduced by Psycharis et al. (2021). This reveals the stages of a person's thinking as they build their understanding of a mathematical concept (Sari et al., 2019; Suarsana et al., 2019). The construction of integer operations in this study is reviewed from the perspective of APOS Theory. In APOS theory, the action stage is the transformation of an individual's perceived object, which is essentially external, so that each step remains explicitly guided by external instructions (Twarog, 2023). The Process Stage is an internal construct resulting from repetitive actions. It is characterized by the ability to make predictions about the solution of a mathematical problem and explain the steps to solve it, as well as the reasons behind them (Aziz, 2021). At the Object stage, one realizes that processes are a totality of actions, and they form these actions into mathematical objects. At this stage, it will be seen how the relationship between objects that have been formed and even actions can be applied to these objects. At the schema stage, individuals have constructed a coordination that links actions, processes, objects, and other schemas to solve a problem. Schemas are coherent, which means that one can connect the components of different schemas when given a problem to decide whether the components fall within the scope of the schema being studied (Novita, 2021; Maharani. In this study, the construction of integer operations at the schema stage is examined through an explanation of the properties of integer operations, as well as the connections between actions, processes, objects, and other schemes that can be used in dealing with integer operation problems.

Integer operation learning still leaves a considerable gap between the curriculum objectives and the reality of student achievement in the field. Students often memorize procedures without understanding the underlying concepts, resulting in superficial knowledge. This is in line with the findings (Huang et al., 2017) This suggests that the low numeracy of the Indonesian people is one reason for their limited understanding of basic mathematical concepts. Various learning innovations, such as the use of technology-based media (Melati et al., 2023) Has not been fully effective in bridging this gap. Dubinsky's developed APOS theory provides a conceptual framework for understanding how students build understanding through actions, processes, objects, and schemas (Kabael, 2011). However, its implementation in the Indonesian education context remains limited. This is the main gap, namely the lack of integration between the practical Gasing method and the systematic theoretical framework of APOS. This gap needs to be addressed through in-depth and practical research, which will enable a significant increase in student understanding.

One solution to overcome this gap is to integrate the Gasing Mathematics method with the APOS theory. The Gasing method, developed by Yohanes Surya, is known as an easy and fun way to learn mathematics. This approach aligns with the constructivist principle, which holds that students construct their own knowledge through meaningful learning experiences. By utilizing the APOS framework, Gasing learning can be directed so that students not only master procedural steps but also understand the relationships between concepts. Additionally, this approach enables students to progress gradually from the concrete to the abstract stage, according to their level of thinking development. Several studies have demonstrated that learning strategies that combine practical activities with theoretical frameworks are more effective in enhancing mathematical understanding (Wijaya, 2020). Therefore, the collaboration between the Gasing method and the APOS theory can be used as a more comprehensive alternative solution. Thus, students are expected to develop a more coherent and in-depth framework for mathematical thinking.

Various previous studies have examined the effectiveness of the Gasing method and the APOS theory separately. The Gasing method enables students to increase their speed and accuracy in completing integer operations. APOS theory is effective in helping students construct a more abstract understanding of mathematical concepts. However, most of these studies only focus on the learning outcome aspect, rather than the thought process that occurs during it. The Gasing method is often viewed as a quick calculation technique rather than a tool for developing in-depth concepts. The integration of cognitive frameworks, such as APOS, with practical methods can strengthen students' abilities to encapsulate and de-encapsulate in mathematical thinking. Thus, there is still room for research that examines how students' thinking processes work when using the Gasing method from the perspective of APOS. This confirms the need for research that fills the gaps in previous studies.

The novelty of this research lies in the integrative approach between the Gasing method and the APOS theory in building an understanding of integer operations. If previous research only highlighted the effectiveness of the Gasing method on learning outcomes, this study focuses on how students' mental structures are formed through the stages of actions, processes, objects, and schemes. This approach offers a new perspective, showing that practical methods are not limited to computational techniques but also serve as a means of constructing more in-depth concepts. In addition, this study fills the gap by exploring the process of encapsulation and de-encapsulation of students when completing integer operations using Gasing in detail. These findings are expected to broaden the research horizon on APOS theory in the context of learning, particularly in relation to local Indonesian culture. Another novelty lies in its contribution to presenting an applicable learning framework, but based on a strong cognitive theory. Thus, this research is not only theoretical but also practical in nature. This novelty makes research more relevant to contemporary challenges in mathematics learning.

The urgency of this research is driven by the need to improve the numeracy literacy of Indonesian students, which is still relatively low, as reported by PISA and several local studies. If not addressed immediately, the low numeracy ability will have a significant impact on the competitiveness of the younger generation in the digital era and global competition. The Gasing method has been proven to attract students to learn mathematics; however, without a deep understanding, the results obtained are only short-term (Arisetyawan & Supriadi, 2020; Bazelais et al., 2018; Maharani, Kholid, et al., 2019; Wu & Yang, 2022). APOS theory provides a conceptual framework that can ensure learning takes place meaningfully and sustainably. Therefore, examining the Gasing method from the perspective of APOS is very important to ensure the effectiveness of learning in terms of cognitive processes. This research also supports the vision of Merdeka Belajar, which emphasizes meaningful learning and is centered on the student. Furthermore, this research is expected to make a significant contribution to the design of innovative learning strategies tailored to the Indonesian context. With this urgency, the research carried out has academic and practical relevance for the world of education.

This study will examine a person's mental structure when constructing their understanding of the integer operation of the casing method. The stages of thinking in APOS theory, namely actions, processes, objects, and schemes, will be analyzed in depth in order to form the right mental structure. Therefore, this study aims to answer the research question: "How is the Construction of Integer Operations of the Top-Mathematics Method from the Perspective of APOS Theory?"

2. METHODS

This study employed a qualitative descriptive approach using a task-based case study design. This approach was chosen because the research aimed to explore in depth how students construct their understanding of integer operations through the stages of APOS (Action, Process, Object, Schema) while engaging in the Gasing Method. Task-based qualitative research allows the researcher to observe and analyze participants' thinking processes as they perform specific mathematical tasks (Agbo et al., 2023). The study was conducted at the Mathematics Education Study Program of the Indonesian Christian University of Toraja. The qualitative case design offers a contextualized understanding of how individual mental constructions emerge within authentic learning activities, rather than in controlled experiments.

The research subjects consisted of three students selected using purposive sampling. The main criteria included (1) students who had participated in formal Gasing Mathematics training, (2) students who demonstrated sufficient mastery of basic integer operations, and (3) students who achieved the highest scores on the integer operation test developed by the researcher. The rationale for selecting the highest-scoring participants was to illustrate a best-case representation—that is, how students who have shown success in using the Gasing Method mentally construct their understanding through APOS stages. This strategy aligns with the principle of *information-rich cases* in qualitative inquiry, which prioritizes depth over breadth to capture the complexity of cognitive processes (Kim et al., 2017).

The instrument used in this study consisted of an Integer Operation Task Sheet based on the Gasing Method. This instrument was designed to elicit students' reasoning about integer operations through realistic problem contexts. Validation of the instrument was conducted by three mathematics education experts, who evaluated its content, construct, and linguistic clarity. Content validity ensured the task aligned with the concepts of integer operations; construct validity confirmed that each item reflected APOS indicators; and language validity verified that the problem statements were clear and comprehensible for participants. The experts provided feedback regarding problem structure, representation of losses and gains, and contextual realism, which were incorporated before the instrument was finalized for data collection.

The story problem used in the task was designed to represent the conceptual construction of integer operations in a meaningful real-life context. The scenario of profit and loss in a building store was chosen because it mirrors everyday experiences that inherently involve positive and negative numbers. Such contexts encourage students to reason about additive inverses and multiple sequential operations, which are core elements of integer understanding (Kahar & Palupi, 2020). The task also aligns with the stages of APOS, allowing researchers to trace how actions (e.g., numerical operations) evolve into processes (mental prediction), objects (generalization of profit–loss relationships), and schemas (integration of multiple operations into coherent structures). Therefore, this problem is considered suitable for capturing students' conceptual development of integer operations.

Data collection utilized method triangulation, combining written task performance with semi-structured interviews. After completing the task, each subject participated in an interview session designed to probe their reasoning and confirm the accuracy of their written responses. The interview guideline consisted of open-ended questions targeting each APOS stage—for instance, prompting subjects to explain how they determined changes in capital (Action), predicted subsequent values (Process), conceptualized the overall pattern (Object), and related it to other arithmetic operations (Schema). Two experts in mathematics education reviewed the interview protocol to ensure the clarity and relevance of questions, thereby strengthening the instrumental validity. Each session was audio-recorded and transcribed verbatim to ensure the accuracy of the data.

The data analysis was conducted qualitatively through the stages of data reduction, data display, and conclusion drawing (Elo et al., 2019). During data reduction, students' written responses and interview transcripts were segmented and coded based on APOS indicators. The coding scheme consisted of four main categories—action, process, object, and schema—with subcodes that described observable indicators, such as manipulation, prediction, encapsulation, and coordination. The coding process followed an iterative cycle of interpretation, verification, and reflection. Triangulation between written and verbal data ensured consistency and reliability of interpretation. Finally, findings were synthesized to describe each participant's mental construction of integer operations within the framework of APOS theory.

3. FINDINGS AND DISCUSSIONS

Findings

The analysis of written tasks and interviews with three participants (S1, S2, and S3) revealed that students' construction of understanding integer operations through the Gasing Method followed the four stages of APOS Theory: Action, Process, Object, and Schema. At the Action stage, all participants solved problems systematically according to instructions, showing procedural fluency through the front-based calculation steps typical of the Gasing approach. During the Process stage, participants began to internalize these procedures, mentally predicting results and explaining the relationships among operations without writing them down. Coding of their verbal responses—such as anticipating carrying and borrowing showed the internalization of repeated actions into cognitive processes. The Object stage appeared when students viewed multiplication as repeated addition and division as the inverse of multiplication, demonstrating encapsulation of mental structures that allowed them to perform mental calculations flexibly.

At the Schema stage, participants were able to integrate all operations and place value concepts into a coherent mental framework, showing coordination among previously formed objects and processes. This indicates that Gasing learning not only develops procedural skill but also fosters conceptual understanding consistent with APOS's reflective abstraction principles (Baye et al., 2021; Suarsana et al., 2019). The findings suggest that the repetitive and structured nature of the Gasing Method effectively supports the transition from external actions to internalized schemas. Thus, integrating the Gasing Method with APOS Theory provides a powerful pedagogical model that bridges procedural fluency and conceptual understanding, aligning with constructivist learning goals and contributing to the development of students' numeracy competence.

Known:

Initial funds = 80,000,000

a. Remaining money:

= 80,000,000 − 25,000,000

= 55,000,000

b. Profit:

= 3 × 55,000,000

= 165,000,000

c. Total profit for 8 months:

3 × 60,000,000 = 180,000,000

66,000,000

88,500,000

72,300,000

80,600,000

77,500,000 +

544,900,000

Total of all money:

55,000,000

165,000,000

544,900,000 +

764,900,000

Will be divided into four people:

764.900.000 ÷ 4 = **191.225.000**

So each person received money of IDR 191,225,000

Figure 1. Subject Written Answers

Based on the written answer of Subject (S) in Figure 1, it can be seen that the subject is in the Action stage, where the task given is completed according to the directions in the question. Starting by writing down what they know, the subject works in detail and sequentially, without skipping any steps. Solve the integer operation problem using the Gasing method completely step by step. This is in line with the previous researcher's opinion that a person is at the stage of action when their thought process is influenced by external stimuli, namely the direction of the problem and the explicit solution step by step (Allsop, 2019; C. da Silva et al., 2020; Hijriani & Simarmata, 2023).

The Process Stage is indicated by the subject when responding orally. Snippets of the interview are presented as follows:

Q: Tell us how you solved this problem.

Q: I first understand the content of the problem so that I can draw up a solution plan.

Q: After understanding the problem, how did you solve it?

Q: I solved these problems in order because each point is interrelated. The core of the question is one located at the end. I start at point A by determining the remaining money after incurring losses. I use the Gasing method of subtraction; the calculation is done from the front. I changed the number 8 to 7 because the subtraction behind it must borrow from the number in front, so the $8 - 2$ The result is 5; thus, until the result is 55 million.

The results of the interview show that the subject is at the process stage when explaining the steps to completing the task. The subject described the process of performing the four integer operations of the gasing method used in solving the problem. The findings in this study align with the opinion of some previous researchers, who suggest that internal control, which results from the repetition of actions performed in a person's mind and expressed through words, is present during the process stage. When explaining, some details are not mentioned, but they still produce the correct answer. The subject performed the calculation using the gas law method in mind and correctly answered the three questions orally. On this basis, the subject is then given several integer operation questions to be answered directly without writing; the results are as follows.

Q: Now, I want you to mention the answers to some of the questions that I will show, just immediately, without writing.

Q: First, 81. Second, 28732263. Third, 525942. Fourth, 1237514. *(Mention each number from the front.)*

These results show that to solve integer operations orally (conjuring) using the casing method, the subjects are at the stage of the thinking process. The subject imagines counting from the front, as in the math principle of the top (Irianti & Wijaya, 2019), to allow the subject to answer the question correctly. From this study, it can be observed that the Process stage in the APOS theory is highly relevant to the primary mission of Gasing mathematics, which is to practice with numerous questions to develop a comprehensive understanding, enabling quick and accurate calculation.

By reviewing the written answers and interview results, we can also identify the stage of the object that occurs during the integer operation of the Gasing method. The following is a snippet of the interview:

Q: How to complete part b?

Q: The result of part a is directly multiplied by 3 by topping as well. Multiplication is done from the front; the result is written while imagining.

Q: What about the calculations in part c?

Q: This is all added up. It starts with sixty million three times, so I multiply it by three and then add up the numbers in order using the top. The calculation is from the back with a cross-out system for results of ten and above.

Q: Well, what is the next process to get the answer to the question?

Q: First of all, all the money that exists must be added up first and then divided by 4. Here I use the gasing method with division from the front. I immediately write the result, and if a division is left, I write it at the top to be combined with the next number to be divided, and so on, until it is completed.

Q: Do you think the four integer operations are interrelated?

S: Yes, they are interrelated. For example, multiplication is equal to the sum of the same number as the multiplier. Then the division is related to reduction, meaning repeated reduction.

When it finds a repeated addition of the same number, the subject writes it in the form of multiplication. Repeated addition is packaged into a multiplication object for further multiplication operations of the top method. This means that in the gasing method, an understanding of the relationship between addition and multiplication is built, and can even re-decompose the form of multiplication into the addition that forms it. The calculation begins from the front, paying attention to the place value of each digit. The calculation process is mostly done without writing because each place value has been packaged in the form of well-structured objects (*encapsulations*) in mind. If needed, the objects can be re-decomposed to be traced for their constituent elements (*de-encapsulation*). For example, when adding from the front, individuals process the numbers according to their place values while predicting the summing results of the next numbers. If the sum of the numbers behind is expected to exceed ten, then the result of the calculation can be increased by one immediately. The same way of thinking also applies to the operations of subtraction, multiplication, and division. This indicates that in integer operations, the gaining mathematical method involves mental processes of encapsulation and de-encapsulation, as described at the object stage in the APOS theory (Rikayanti, 2017).

Based on the stages of action, processes, and objects represented by the subject, a scheme can be formed when constructing integer operations with the Gasing mathematical method. Broadly speaking, the schema of the subject, as it relates to constructing his understanding of integer operations, is illustrated in Figure 2.

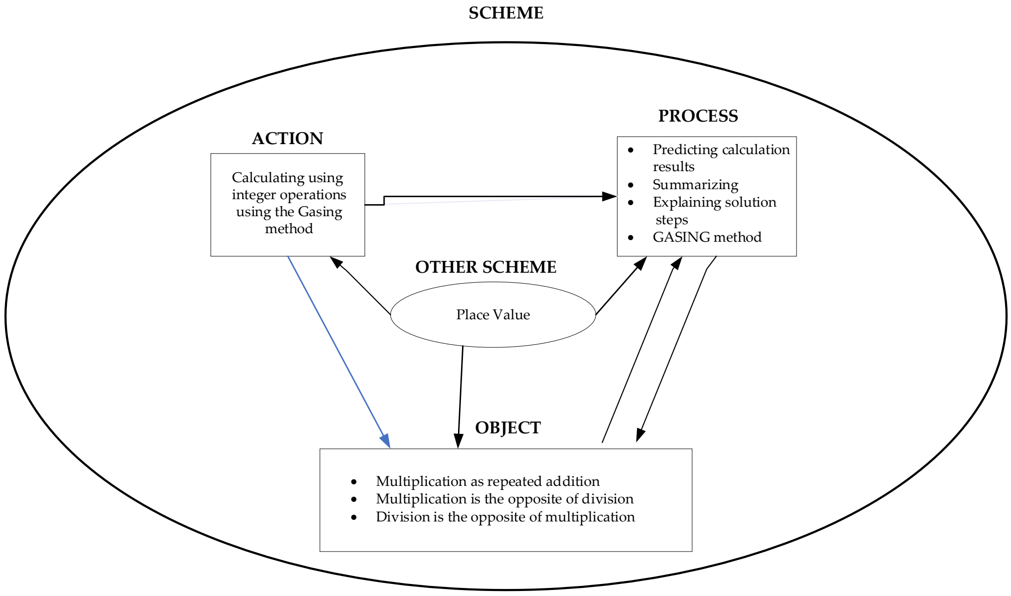

Figure 2. Construction Scheme of Integer Operation Gasing Method

Based on the scheme in Figure 2, it can be stated that the Action stages, as shown through the calculation of integer operations using the Gasing method, are applied to each stage. This supports the statements of some previous researchers who argue that action is the first step in building mathematical understanding and is employed at the stage of the process, object, and scheme of the subject to solve problems by creating a plan, i.e., solving sequentially.

Discussion

The findings of this study reaffirm that the stages of Action, Process, Object, and Schema in APOS Theory can effectively describe the cognitive development of students learning integer operations through the Gasing Method. Consistent with previous studies, structured and repetitive actions indeed facilitate the internalization of procedural understanding. However, unlike those studies that emphasize procedural mastery alone, this research highlights how the Gasing Method facilitates the transition toward conceptual encapsulation, particularly when students begin to recognize relationships among operations. This suggests that Gasing's "front-based counting" pattern serves not only as a computational aid but also as a scaffolding mechanism for abstraction. Nevertheless, these results should be interpreted cautiously—students' rapid calculation ability does not automatically imply deep conceptual flexibility, as APOS theory demands iterative reflection between concrete and abstract stages.

While the Gasing Method proves effective in improving comprehension of integer operations, its scope of applicability remains limited. This method's emphasis on concrete visualization and procedural repetition may be less suited for abstract mathematical domains such as algebraic reasoning, proof, or calculus, where symbolic generalization dominates. Moreover, the small number of participants and the qualitative case design limit the generalizability of the findings beyond similar contexts. Thus, rather than claiming broad curricular implications, this study offers contextual insights for developing micro-level pedagogical models that integrate local methods with cognitive frameworks. Future research should expand participant diversity and explore how Gasing APOS integration functions in more complex mathematical topics. In this way, the present study positions itself as a preliminary exploration that enriches the discourse on linking culturally grounded pedagogies with cognitive learning theories, rather than as a definitive policy recommendation.

The results of the study showed that at the *action stage*, the subjects solved integer operation problems using the Gasing method completely and systematically. Each step is carried out according to the instructions of the question, making it clear that external stimuli influence thinking. APOS theory, the action stage is an initial transformation that is explicit and requires external direction (Psycharis et al., 2021). This is in accordance with the fact that students are not able to generalize without question guidance. The presence of the easy-to-practice Gasing method strengthens the action process because simple step patterns help students. The process of quick counting without writing also encourages students' basic numeracy skills. Thus, the action stage becomes a crucial foundation for a more complex construction of thought.

The *process stage* in this study is demonstrated when the subject can explain the completion steps orally without needing to write down the entire procedure. This condition indicates that students are beginning to develop an internal representation of the results of repeating the actions carried out. Processes are formed when actions have been internalized into the individual's mind. This demonstrates the presence of internal control in mathematical thinking. The Gasing method, which emphasizes counting patterns from the front, provides students with opportunities to develop mental strategies for solving problems. These findings support the view Syafarina & Zaenuddin (2025) The repetition of actions using structured methods can foster the ability to predict and engage in mathematical reasoning; therefore, the process stage is a marker of transition towards conceptual understanding.

At *the object stage*, the study found that students were able to see the relationships between operations, such as multiplication as a form of repeated addition. This encapsulation ability is a characteristic of the object stage according to the APOS theory. The subject can convert the addition operation into multiplication, and vice versa, so that the mathematical concepts are more fully intertwined. This process of encapsulation and de-encapsulation shows that students are no longer fixated on mechanistic procedures. The Gasing method supports this process by emphasizing the calculation based on the value of the place from the front. Thus, students' understanding develops in a more abstract direction. This object stage is important as a bridge before students construct a more comprehensive scheme.

The *schema* stage is achieved when students integrate actions, processes, and objects in a coherent and unified manner. The subject compiles a complete problem-solving plan involving four interrelated integer operations. Schemas allow individuals to connect the various cognitive representations they have. In this study, the subject scheme is illustrated through an understanding of the relationship between multiplication and division, as well as mastery of place values. This proves that Gasing's method not only emphasizes quick calculations but also facilitates the formation of mathematical frameworks. The schema formed becomes the basis for problem-solving flexibility. In other words, the schema stage shows the culmination of the student's cognitive construction in learning integer operations.

The findings of this study confirm the importance of integrating practical approaches with theoretical frameworks in mathematics learning. The Gasing method provides concrete activities, while the APOS theory provides a conceptual foundation for cognitive development. Effective learning occurs when there is an interaction between concrete experience and the internalization of abstract concepts. (Alexander, 2019). Thus, the fun Gasing method can be combined with APOS to produce a deeper understanding. The integration of the two has the potential to increase student numeracy significantly. Therefore, this research makes a theoretical and practical contribution to the development of learning strategies.

The quick numeracy skills generated through Gasing's method do not stand alone, but rather form part of a broader framework for thinking and learning. The study's results show that students who practice with this method have an easier time developing internalization processes. This supports the idea that cognitive development occurs gradually from the concrete to the abstract. Gasing plays a role in providing concrete experiences, while APOS explains the abstraction stage. That way, this method can address criticisms of mathematics learning that often emphasize mechanical procedures. Speed of counting is a side effect of mastering more in-depth concepts. Therefore, this method is relevant for building sustainable numeracy.

The discussion also revealed that the Gasing method plays a significant role in reducing mathematics anxiety among students. Fun activities, such as speed counting games, motivate students to study without feeling pressured. Math anxiety can hinder logical thinking processes and lower academic performance. Thus, the attractive Gasing method helps students be more confident. This process also supports the stages of action to schema in APOS theory because students are more open to concept exploration, gaining's role in building motivation to learn mathematics. So, Gasing-APOS-based learning can integrate affective and cognitive aspects in a balanced manner.

The findings of this study also have implications for the development of the Independent Learning curriculum. The curriculum emphasizes contextual, fun, and meaningful learning. The Gasing method, combined with the APOS theory, aligns with this spirit because it provides students with space to build knowledge independently. Emphasizing the importance of *a spiral curriculum* where concepts are taught repeatedly with an increased level of abstraction (Borji et al., 2018; Education et al., 2018; Planell & Trigueros, 2019). The Gasing-APOS integration offers a natural spiral mechanism that encompasses the stages of actions, processes, objects, and schemas. Thus, this strategy can serve as a reference for developing innovative learning tools. The study's results demonstrate that this approach can enhance the quality of student learning. Therefore, the implications are relevant for national education policy. From the perspective of previous research, these results provide new confirmation. The effectiveness of Gasing in improving learning outcomes shows the strength of APOS in constructing concepts (Kandemir et al., 2020; Salgado & Trigueros, 2015).

4. CONCLUSION

This study concludes that integrating the Gasing Method with APOS Theory provides a coherent framework for understanding how students construct integer operation concepts from procedural to conceptual levels. The findings reveal that the structured and sequential nature of Gasing learning aligns with the action–process–object–schema stages, thereby transforming repetitive computation into meaningful abstraction. This integration offers a conceptual bridge between practical pedagogy and cognitive theory, highlighting that numeracy skills can grow not only through mechanical practice but also through reflective understanding. For mathematics teachers, this research emphasizes the importance of designing learning that connects local, enjoyable practices with theoretical depth to strengthen both student motivation and reasoning. For students, it provides evidence that rapid calculation ability emerges as a byproduct of deeper conceptual coherence rather than mere memorization. The study also contributes to the refinement of the Gasing Method as a culturally grounded yet theoretically informed model that supports Indonesia's effort to improve numeracy literacy. However, the limited number of participants, single-instrument data, and descriptive analysis restrict the generalizability of the findings. Future studies are encouraged to focus on specific operations or larger samples to deepen understanding of how Gasing APOS integration operates across mathematical domains. Ultimately, the uniqueness of this study lies in its conceptual synthesis, which demonstrates that culturally rooted instructional innovations can be aligned with global cognitive learning theories to foster meaningful and sustainable mathematics learning.

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